Gamma–ray spectroscopy

Part VI

Nuclear Lifetimes and Moments Measurement
Objectives

- **Lifetime measurements**
  - bound excited nuclear states
  - determination of absolute transition matrix elements
  - reduced transition probabilities
    - $B(E2;2^+ \rightarrow 0^+)$ – ground state deformation
    - collectivity of exited states
    - shape coexistence

- **Nuclear moments measurements**
  - sensitive probes of the structure of the nucleus
  - magnetic moments
    - sensitive to the single particle configurations (mixing)
  - quadrupole moments
    - sensitive to collective properties (polarization, deformation, effective charges)

\[ \mu = g \tilde{I} \]

\[ Q = e \sum_{i=1}^{A} (3z_i^2 - r^2) \]
- **Excited nuclear states**
  - remain excited for a lifetime $\tau$

  \[
  \Gamma \tau = \hbar
  \]

  \[
  \Gamma = \frac{\hbar}{\tau} = \frac{6.58 \times 10^{-22} \text{MeV} \cdot s}{\tau}
  \]

  $\tau > 10^{-22} \text{s}$ $\rightarrow$ width becomes smaller than the separation between states

  \[
  \Gamma \propto \left| \langle \psi_f | M | \psi_i \rangle \right|
  \]

  where $M$ is the operator for the decay and $\psi_i$, $\psi_f$ are the wavefunctions of the initial and final states

  if more decay modes are concurrent:

  \[
  \Gamma = \sum_{i=1}^{n} \Gamma_i
  \]
Transition Probabilities

Nuclear lifetimes $\rightarrow$ transition probabilities

$$T_{fi}(\lambda L) = \frac{8\pi (L + 1)}{\hbar L ((2L + 1)!!)} \left( \frac{E_\gamma}{\hbar c} \right)^{2L+1} B(\lambda L; J_i \rightarrow J_f)$$

$B(\lambda L; J_i \rightarrow J_f)$ = reduced transition probability

$$T = \frac{1}{\tau} \left( \text{s}^{-1} \right) \quad \tau = \text{lifetime,} \quad E_\gamma \ (\text{MeV}) = \text{gamma ray energy}$$

$$T(E1) = 1.587 \cdot 10^{15} E_\gamma^3 B(E1) \quad T(M1) = 1.779 \cdot 10^{13} E_\gamma^3 B(M1)$$
$$T(E2) = 1.223 \cdot 10^9 E_\gamma^5 B(E2) \quad T(M2) = 1.371 \cdot 10^7 E_\gamma^5 B(M2)$$
$$T(E3) = 5.698 \cdot 10^2 E_\gamma^7 B(E3) \quad T(M3) = 6.387 \cdot 10^7 E_\gamma^3 B(M3)$$

B is given in units of $\mu_N \cdot \text{fm}^{L-1}$
Nuclear Lifetimes

A collection of $N_0$ nuclei produced at the time $t=0$ will decay with a mean lifetime $\tau$ according to the law:

$$N(t) = N_0 e^{-t/\tau}$$

where $\tau = 1/\lambda$ with $\lambda$ the transition probability (decay rate).

$T_{1/2} = 1.5, \tau = T_{1/2}/\ln(2) = 2.16$

The half–live

$T_{1/2} = \tau \ln 2$
Quadrupole Deformation

- Collective E2 transitions

\[ B(E2) = \frac{5}{16\pi} Q_0^2 \left| \left\langle J_iK20 | J_fK \right\rangle \right|^2 \]

\[ Q_0 = \frac{3}{\sqrt{5\pi}} Z R^2 \beta_2 \left( 1 + \frac{1}{8\pi} \beta_2 + \frac{5}{8\pi} \beta_2^2 + \ldots \right) = \text{intrinsic quadrupole moment} \]

\[ \left\langle J_iK20 | J_fK \right\rangle = \sqrt{\frac{3(J - K)(J - K - 1)(J + K)(J + K - 1)}{(2J - 2)(2J - 1) J (2J + 1)}} \]

\[ = \text{Clebsch–Gordon coefficient} \]

\[ \frac{1}{\tau} = 1.223 \cdot E_\gamma^5 \frac{5}{16} Q_0^2 \left| \left\langle J_iK20 | J_fK \right\rangle \right|^2 \]

\[ B(E2) = \frac{0.081642 \cdot B_i}{\tau (ps) \left[ E_\gamma (\text{MeV}) \right]^5 (1 + \alpha)} \]
Feeding of Excited States – Lifetimes

the observed lifetime of a state depends on the lifetimes of the states feeding it

\[ \text{apparent lifetime} \]

the relation between apparent lifetime and intrinsic lifetime of the levels is given by the Bateman equations of radioactive decay

\[
\frac{dN_i(t)}{dt} = \sum_{j=i+1}^{M} N_j(t) \lambda_j - N_i(t) \lambda_i
\]

where \( N_i(t) \) is the population of level \( i \) at time \( t \)

assuming \( N_{M-1}(0) = N_{M-2}(0) = \ldots = N_1(0) = 0 \)

\[
N_i(t) = N_0 \sum_{j=i+1}^{M} c_j e^{-\lambda_j t}
\]

\[
c_i = \frac{\prod_{j=i+1}^{M} \lambda_j}{\prod_{j=i+1}^{M} (\lambda_j - \lambda_i)}
\]
Feeding of Excited States – Lifetimes

\[ N_0 \]

Blue: parent \( T_{1/2}=1.5, \ N_1(0) = 100\% \)
Red: daughter \( T_{1/2}=3, \ N_2(0) = 0\% \)

Number of atoms [%]

Time [arb.]
Feeding of Excited States – Lifetimes

The observed lifetime of a state depends on the lifetimes of the states feeding it

\[ \tau_\text{apparent} \]

...the relation between apparent lifetime and intrinsic lifetime of the levels is given by the Bateman equation of radioactive decay

\[
\frac{dN_i(t)}{dt} = \sum_{j=i+1}^{M} N_j \lambda_j - N_i \lambda_i
\]

where \( N_i(t) \) is the population of level \( i \) at time \( t \)

If there is a side feeding:

\[ N_{M-1}(0) \neq 0; N_{M-2}(0) \neq 0; \ldots; N_1(0) \neq 0 \]

\[
N_i(t) = N_0 \sum_{j=i+1}^{M} c_j e^{-\lambda_j t} + \sum_{j=1}^{M-1} N_j^{SF} (t) \cdot c_j^{SF} e^{-\lambda_j^{SF} t} \sum_{j=i}^{M-1} c_j e^{-\lambda_j t}
\]
the Bateman equation is not a single equation but a method for setting up a differential equations describing the decay of the chain of interest as a function of time based on the decay rates and the initial population of the states

Side feeding

If there is a side feeding:

\[ N_M(0) \neq 0; N_{M-2}(0) \neq 0; \ldots; N_1(0) \neq 0 \]

\[ N_i(t) = N_0 \sum_{j=i+1}^{M} c_j e^{-\lambda_j t} + \sum_{j=i}^{M-1} N_j^{SF}(t) \cdot c_j^{SF} e^{-\lambda_j^{SF} t} \sum_{j=i}^{M-1} c_j e^{-\lambda_j t} \]
Feeding of Excited States – Lifetimes

in case of fusion–evaporation reactions large side feeding from unresolved states in the continuum with unknown lifetimes

the feeding times from such states are usually much shorter than the in band decay times
Feeding of Excited States – Lifetimes

in case of fusion–evaporation reactions large side feeding from unresolved states in the continuum with unknown lifetimes

the feeding times from such states are usually much shorter than the in band decay times

side feeding effects on the lifetime measurements can be avoided by using coincidence sets on transitions above the state of interest
Feeding of Excited States – Lifetimes

\[ N(t) = 70 \times \exp\left(-\frac{t}{\ln(2)1.5}\right) + 30 \times \exp\left(-\frac{t}{\ln(2)15}\right) \]
Techniques for Lifetime Measurement

Direct Methods
- determine directly $\tau$

Indirect Methods
- determine the width $\Gamma$

GRID: Gamma ray induced Doppler broadening
DSAM: Doppler shift attenuation method
RFD: Recoil straggling method
RDDS: Recoil distance Doppler shift method
RSAM: Recoil shadow anisotropy method
FEST: Fast electronic scintillation method
NFR: Nuclear resonance fluorescence
Coulex: Coulomb excitation cross section
Irradiation and Counting

Lifetimes ~ sec – min
- sample irradiated to produce isomeric state
- fast transport system: rabbit or gas jet
- counting decays with a Ge detector

rabbit system: 3 m in 0.5 s

D.F. Geesaman et al., PRC19(1979)1938
Irradiation and Counting

Lifetimes ~ sec – min
- sample irradiated to produce isomeric state
- transport sample to a low–background area (tape transport)
- counting decays with a Ge detector

\[ ^{39}\text{Ca} \text{ produced in REXTRAP at ISOLDE} \]

\[ T_{1/2} = (444.05 \pm 2.31) \text{ ms} \]

B.Blank et al., EPJ A44 (2010)363
Irradiation and Counting: FEST

- poor Ge time resolution limit $\tau >$ several ns
- LaBr$_3$ detectors excellent time resolution (< 400 ns)
- Ge–(LaBr$_3$–LaBr$_3$) coincidences

$^{169}\text{Tm}(^{12}\text{C},7\text{n})^{174}\text{Re} \rightarrow ^{174}\text{W}^*$

$\beta$

the slope yields directly the lifetime

V.Werner et al., J. of Physics312(2011)092062
In-beam FEST with LaBr$_3$ Detectors

- Ge–(LaBr$_3$–LaBr$_3$) coincidences
- LaBr$_3$ detectors excellent time resolution – fast decay time (26 ns)
  → better than 400 ps
- relatively good energy resolution – large photon yield (63 photons/keV)
  → 2.6% @ 662 keV ($^{137}$Cs)

Detectors @ LNL

BrilLanCe 380 Integrated Design
Model 51S51/B380/PMT2,2

- Crystal 2"x 2"
- XP5500B 8–stages 2.2" PMT
- AS20 Voltage Divider

Performance (data sheet)
- PHR@662keV ≈ 2.6%
- CRT@511keV ≈ 0.45 ns

Connectors
- 1 output from Anode (DC coupl.)
- 1 negative HV supply
In–beam FEST with LaBr$_3$ Detectors

Typical energy spectra

- $^{60}$Co, $^{88}$Y, $^{137}$Cs

- Int. activity

Good agreement with data sheet specifications

Typical time spectra

$^{88}$Y $\gamma$–$\gamma$ coinc.

$E_\gamma$ [keV]

- 789 + $\beta^-$
- 1436 + 32 keV X–rays

Time [a.u.]

- $^{88}$Y $\gamma$–$\gamma$ coinc.

- $^{138}$Ba
- $^{138}$Ce

32 keV X–rays ($K_{\alpha 1}$)
In–beam FEST with LaBr₃ Detectors

**Energy–Time Correlation**

- **Time**: 0–2 ns
- **Energy**: 0–2000 keV

**Eγ–Eγ–Δt Matrices**

- **23/2⁻**: 3114.4 keV
- **19/2⁻**: 2158.4 keV
- **15/2⁻**: 1360.3 keV
- **11/2⁻**: 845.5 keV
- **7/2⁺**: 640.5 keV

**107 Cd**

- **956.0**: Ge gate for 7/2⁺
- **798.1**: Ge gate for 5/2⁺
- **514.8**: Ge gate for 11/2⁻

**Gate Setting**

- **1332 keV**

**Time Calibration**

- **0.69(3) ns**: Half-life of the transition

**Fit Prompt Gaussian**

- **+ exponential decay**

**Delayed Spectrum**

- **States in LaBr₃**

**LaBr₃ Detector**

- **Anode**
- **Dynode**

**TAC**

- **Start**
- **Stop**

**Energy Delay**

- **Long delay**
- **Short delay**

**LaBr₃ Time**

- **Multiplicity**

**AND**

- **Ge multiplicity ≥ 1**

**Gate & delay gener.**

**Counts**

**Prompt Coincidence Spectrum**

- **205.0 keV**
- **956.0 keV**
- **640.5 keV**

**Delayed Spectrum**

- **205.0 keV**
- **798.1 keV**
- **515.0 keV**

**Half-life**

- **T₁/₂ = 0.69(3) ns**

**Time Walk Correction**

- **5 ps**

**References**

- **N.Marginean et al., EPJ A46 (2010)329**
Pulsed Beam Technique

- width of the beam pulse
  \[ < \tau \ (\sim \text{ns}) \]
- beam repetition rate
  \[ 100 \text{ ns} - \mu \text{s} \]
- record spectra in between the pulses
  low background
- exponential decay of the isomer during the no–beam period

J. Wrzesinski et al., EPJ A10 (2001) 259
Methods Based on the Doppler Effect

- relativistic Doppler shift

\[ E_{sh}(\theta) = E_0 \frac{\sqrt{1 - \beta^2}}{1 - \beta \cos \theta}; \quad \beta = \frac{v}{c} \]

- in the case of the usual fusion–evaporation reactions \( \beta \ll 1 \) (few \%) so that

\[ E_{sh}(\theta) \approx E_0 (1 + \beta \cos \theta) \]

- allows to distinguish gamma rays emitted by stopped or in–flight nuclei

- Doppler shift of the gamma rays allow for the determination of the emitting nuclei velocity

\[ \beta \approx \frac{E_{sh}(\theta) - E_0}{E_0 \cos \theta} \]

- Doppler shift lifetime measurement methods

  - Recoil Distance Doppler Shift (RDDS) \( 1 \) ps – 1 ns
  - Doppler Shift Attenuation Method (DSAM) 100 fs – 1 ps
  - Fractional Doppler Shift Method \( 5 \) fs – 50 fs
Recoil Distance Doppler Shift (RDDS) Method

- thin target \( \sim 500 \, \mu\text{m/cm}^2 \) (movable)
- thick stopper (‘plunger’ foil) usually in Au
- recoils decay in-flight over the distance \( x \)
- recoils decay as stopped in the stopper
- both stopped and in-flight emitted gamma rays are detected
  - \( \text{sh} \): shifted component
  - \( \text{u} \): unshifted component

\[
E_{sh}(\theta) \approx E_u \left(1 + \beta \cos \theta \right)
\]

Measure the difference of intensity of the two components as a function of the target–to–stopper distance
Recoil Distance Doppler Shift Method

\[ 2^+ \rightarrow 0^+ \quad 4^+_1 \rightarrow 2^+_1 \quad 6^+_1 \rightarrow 4^+_1 \]

\( \theta = 35^\circ \)
Recoil Distance Doppler Shift Method

- intensity of the in–flight component
  \[ I_{sh} = N_0 \left( 1 - e^{-\frac{x}{v_{rec}\tau}} \right) \]

- Intensity of the stopped component
  \[ I_u = N_0 e^{-\frac{x}{v_{rec}\tau}} \]

- the ratio of the unshifted component to the total intensity
  \[ R = \frac{I_u}{I_u + I_{sh}} = e^{-\frac{x}{v_{rec}\tau}} \]

depends on the lifetime \( \tau \)

Decay curve

\[ R \]

W.Scmitz et a., PLB 303(1993)230
Recoil Distance Doppler Shift Method

- precision of the distance $\sim 0.1 \, \mu m$
- target–to–stopper distance measured as a capacitance between the 2 plates
- well stretched target and stopper and good parallelism
- distance scale transformed in time scale from average velocity

- in complex level schemes the result is altered by
  - multiple exponential decay components
  - feeding from states above with comparable lifetimes
- use Bateman for the feeding of the states and make a global fit of the level scheme
  - doesn’t work for unknown feeding
Differential Decay Curve Method

- the Bateman equation can be reformulated in terms of the observed unshifted intensity for different target–to–stopper distances

\[
\tau_i \frac{dN_i(t)}{dt} = \sum_j b_{ji} N_j(t) - N_i(t)
\]

where \( b_{ji} \) are the branching ratios for the feeding transitions

Coincident Differential Decay Curve Method

gate from above – eliminates the problems of unknown feeding

– selects the feeding path of the states

\[
I_{BA}^{BA} = I_{ss}^{BA} + I_{su}^{BA} + I_{uu}^{BA}
\]

\[
I_{us}^{BA} \quad \text{it is not possible}
\]
Coincident DDCM

- gating from above on B (A is measured)
- gate on (s+u) \( \rightarrow \) eliminates background & side feeding
- gate on u \( \rightarrow \) direct lifetime measurement with no need of solving the Bateman equation

\[
\tau = \frac{I^{BA}_{su}(x)}{v_{rec} \frac{d}{dx} I^{BA}_{ss}(x)}
\]

\( ^{122}\text{Ba} \)

\( 6^{+}_i \rightarrow 4^{+}_i \)
Differential RDDS Method

- for use with in setups that require the detection of the recoiling nuclei
- stopper replaced with a degrader

CLARA & AGATA + PRISMA
Differential RDDS Method

J.J. Valiente-Dobon et al, PRL102(2009)242502
Differential RDDS Method

Differential RDDS AGATA measurement at LNL with $^{76}$Ge + $^{238}$U reaction

Spectra taken at 50 kHz per capsule counting rate

AGATA-PRISMA RDDS measurement $\theta_G=55^\circ$

E. Sahin, A. Görgen, M. Doncel

Plunger IKP University Köln
TU Darmstadt
Doppler Shift Attenuation Method (DSAM)

- thin target ~500 μm/cm²
- target backing of a high Z stopper material (Au/Pb/Ta)
- recoils decay in–flight while stopping in the target
- lineshape depends on the nuclear lifetime

$v(t)$ from Monte Carlo simulation with stopping power

Forward Angle

Full Shift (short lifetime)

No Shift (long lifetime)
Doppler Shift Attenuation Method (DSAM)

- DSAM applies when the lifetime is of the order of the slowing down time of the recoils in the target ($10^{-13} - 10^{-12}$ s)

- analysis of the gamma ray lineshapes as a function of the detection angle to determine the nuclear lifetime

\[ E_{sh}(\theta) \approx E_0 \left(1 + \beta \cos \theta \right) \]

- the lineshape will depend on the velocity of the recoil (from $v_0$ to 0) when the gamma ray was emitted

- the centroid of the lineshape is a measure of the average recoil velocity

- the centroid is expressed usually in terms of Doppler Shift Attenuation Factor

\[ F(\tau) = \frac{\bar{V}}{V_0} = \frac{1}{V_0 \tau} \int_0^{\infty} v(t) e^{-\frac{t}{\tau}} dt \]
Doppler Shift Attenuation Method (DSAM)

- \( v(t) \) is determined based on the stopping powers

- stopping powers:
  - electronic stopping
  - nuclear stopping (scattering of the nucleus) – Monte Carlo simulations of the recoil velocity profiles

- the lineshape \( dN/dE \) can be directly related to the nuclear lifetime

\[
dN(t) = \frac{1}{\tau} e^{-\frac{t}{\tau}} dt
\]

- a knowledge of the velocity distribution \( dv/dt \) allows for the calculation of the lineshape for a given \( \tau \)

- the accuracy of the method is limited by the
  - inaccuracy of the stopping power models (10 – 15% systematic error)
  - unknown side feeding of the states
Steps for the simulation of the lineshapes

- simulate the slowing down history of the recoils in the backing; provides \( v(t) \) and \( \theta_R(t) \)
- calculate the Doppler shift observed at the angle \( \theta_\gamma \) as a function of time
- calculate the population \( N(t) \) of the state by solving the Bateman equation
- simulate the energy spectrum in a gamma ray detector from \( N(t) \)
- compare with the experimental lineshape and minimize \( \chi^2 \)
Doppler Shift Attenuation Method (DSAM)

\[ \Theta = 72^\circ \]

\( E_\gamma \) (keV)  

\( 52^{\text{Fe}} \) 

Gate from below
Fractional Doppler Shift

- in the case of superdeformed bands with large $Q_t$ lifetimes are very short (<100 fs)
- gamma ray emission occurs before significant slowing down of the recoils
  → large Doppler shift with angle and no significant lineshape
- use of $F(\tau)$ for the analysis of the centroids shift

$^{48}\text{Ti (}@\text{214 MeV)} + ^{100}\text{Mo} \rightarrow ^{144}\text{Gd}$

target $^{100}\text{Mo} \ 1 \ \text{mg/cm}^2 \ \text{on Au} \ 10 \ \text{mg/cm}^2$

GASP – configuration I
Fractional Doppler Shift

- in the case of superdeformed bands with large \( Q_t \) lifetimes are very short (<100 fs)
- gamma ray emission occurs before significant slowing down of the recoils
  \( \rightarrow \) large Doppler shift with angle and no significant lineshape
- use of \( F(\tau) \) for the analysis of the centroids shift

\[
F(\tau) = \frac{\bar{V}}{V_0} \implies \bar{V} = F(\tau) V_0
\]

Centroid shift

\[
E_c(\theta) \approx E_0 \left( 1 + F(\tau) \beta_0 \cos \theta \right)
\]

\[
F(\tau) = \frac{E_c - E_0}{E_0 \beta_0 \cos \theta}
\]
Krakow RFD (straggling) Method

$^{18}\text{O} + ^{30}\text{Si} @ 68\text{MeV}$

$\tau = 50\text{fs} - 1\text{ps}$

Target

Position sensitive detector

$^{18}\text{O}^{(7}\text{Be,3n})^{12}\text{C}$

$1.0 \text{ mg/cm}^2$

$0.5 \text{ mg/cm}^2$

$0.25 \text{ mg/cm}^2$

$0.1 \text{ mg/cm}^2$

TRIM Monte Carlo calculation

J.F.Ziegler J.P.Biersack

P.Bednarczyk, W.Meczynski, J.Styczen et al.
A short lifetime determination with RFD

\[ R = \frac{A}{A+B} \]

68MeV \(^{16}\text{O}\) + 0.8mg/cm\(^2\) \(^{30}\text{Si}\); 
Recoil transit time \(\approx 0.4\) ps

\[ R = v [1 - \exp (-v\tau)] \]

\( \tau > T \)

\( \tau < T \)

The range of measured lifetimes can be chosen by a selection of the target thickness.

\( \tau \approx 40, 200, > 4 \text{ ps} \)

In the measurement \( \tau \) ranging from 40 to 800 fs could be determined.

P.Bednarczyk, W.Meczynski, J.Styczen et al.